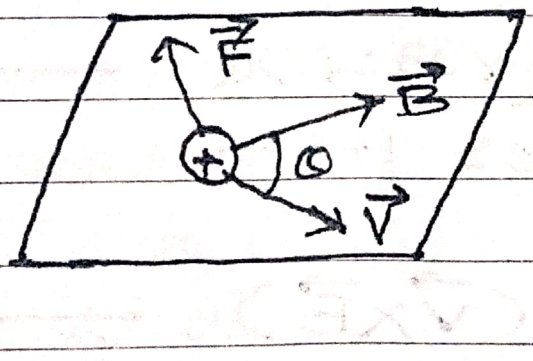
3 Magnetostatics

Magnetic Field

The space around magnet or current carrying conductor in which its magnetic effect can be experienced, is called magnetic field.

It is a vector field. Its magnitude and direction at any point are specified by $\vec{B}$. It is found experimentally that whenever moving charge particle having charge q enter ~~into~~ in a magnetic field $\vec{B}$ with a velocity $\vec{v}$ it experience a force $\vec{F}$. Fig.



The magnitude F of this force is directly proportional to

a charge on the particle q

b velocity v of the particle

c magnetic field B and $\sin\theta$ where θ is the angle between $\vec{V}$ and $\vec{B}$

combining these factor we got

$$F \propto qVB\sin\theta$$

$$F = KqVB\sin\theta$$

where K is constant of proportionality. Its value is found to be 1 i.e K=1.

$$\therefore F = qVB\sin\theta \quad \text{———} (1)$$

or in vector form

$$\vec{F} = q(\vec{V} \times \vec{B}) \quad \text{———} (2)$$

this equation is called Lorentz force experience by moving charge particle in the magnetic field.

~~It is~~ In eqn (1) if $q=1$ $v=1$ $\theta = 90^\circ$ then

$$F = B \quad \text{------} \quad (3)$$

Thus magnetic field at a point is equal to the force acting on a unit charge involving with unit velocity in a direction at right angle to magnetic field.

It is clear from the eqn (2) that F is directed in a direction perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.

~~SI unit of $\vec{B}$ is a tesla and is denoted by T~~

If $\vec{v}$ and $\vec{B}$ are mutually perpendicular

$$F = qvB\sin 90^\circ$$

$F = qVB$ —— (4)

It means max^{m} value of Force experience by moving charge in a direction perpendicular to the plane containing $\vec{V}$ & $\vec{B}$

SI unit of $\vec{B}$ is called tesla and is denoted by T.

We obtain from equation (4)

$$B = \frac{F}{qV}$$

$F = 1N \quad q = 1C \quad V = 1\,m/s$

then $B = 1T$

$$1T = \frac{1N}{1C \times 1\,m/s}$$

$$= \frac{1N}{1A \times 1m}$$

$$= 1\,N\,A^{-1}\,m^{-1}$$

1 tesla is also known as $\frac{1\ \text{Weber}}{\text{meter}^2}$ $\qquad \frac{C}{S} = A$

CGS. $1\,T = 10^4$ Gauss

Dimension of B

$$F = q \times B$$

$$B = \frac{F}{qV} = \frac{M^1 \cancel{L^1} T^{-2}}{A^1 T^1 \cancel{L^1} T^{-1}}$$

$q = I \times t$ $\qquad\qquad$ $V = \cancel{x}/T$

$= A^1 T^1$

$acc^n = \frac{vel}{time} = \cancel{\frac{L/T}{t}}\ \frac{L}{T} \cdot \frac{1}{T} = L T^{-2}$

$$B = M^1 A^{-1} T^{-2}$$

Boit - Savart's law

- only valid for very very small element

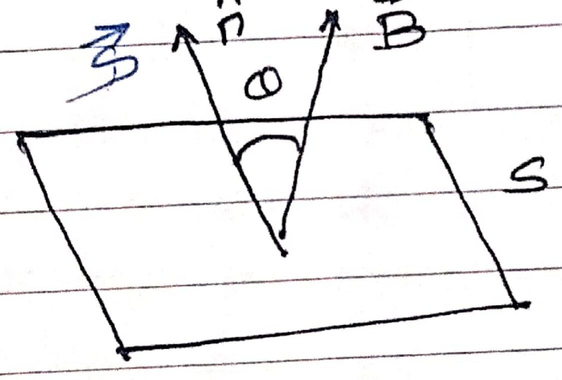
- This law is analogous to coulomb's law in electrostatics

1)

Magnetic flux (ϕ_B)

Magnetic flux (ϕ_B) through a given surface is defined as the total no. of magnetic lines of induction passing through that surface.

If a plane surface S is held in a uniform magnetic field $\vec{B}$ then magnetic flux through area S is



$$\phi_B = BS\cos\theta$$
$$= \vec{B}\cdot\hat{n}S$$
$$= \vec{B}\cdot\vec{S}$$

where θ is the angle between the direction of magnetic induction $\vec{B}$ and unit vector along the normal to the surface area S

SI unit - weber

CGS unit of ϕ_B is Maxwell.

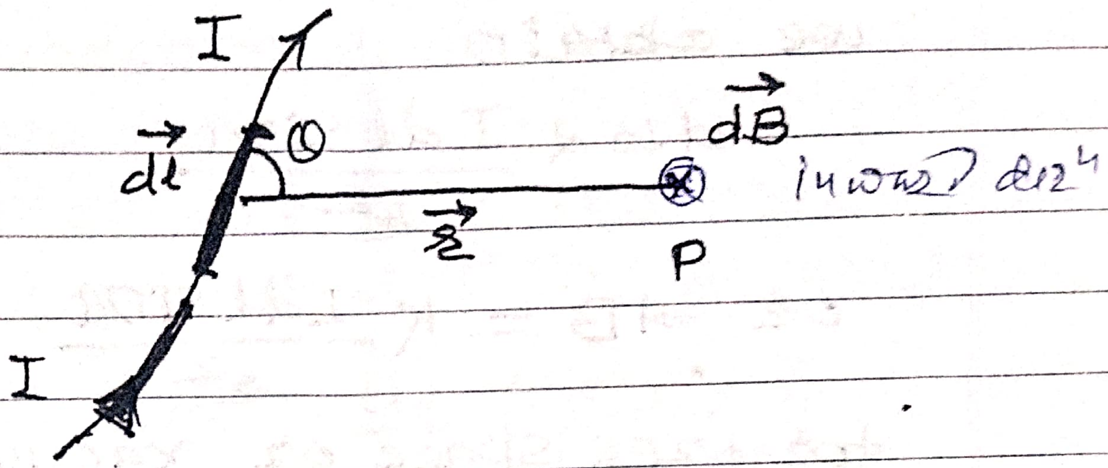
1 weber = 10^8 maxwell.

dimension of the flux

$$\phi_B = B \times S$$

$$= M^{1} A^{-1} T^{-2} \cdot L^{2}$$

Boit - Savart's law



suppose a small element of length dl of a conductor through which a current I is flowing.

Let $\vec{dB}$ the small magnetic induction due to small element $\vec{dl}$ at a point p having position vector $\vec{r}$ from the given element. Let θ be the angle between $\vec{dl}$ and $\vec{r}$

Then accourding to Boit-Savart's law.

1- $dB \propto I$

2 $dB \propto dl$

3 $dB \propto \sin\theta$ and

4 $dB \propto \frac{1}{r^2}$

On combining all these factors, we obtain

$$dB \propto \frac{I\,dl\sin\theta}{r^2}$$

or $$dB = K\frac{I\,dl\sin\theta}{r^2}$$

for free space or vacuum in SI unit the value of constant K is given by

$$K = \frac{\mu_0}{4\pi} = 10^{-7}\,\frac{Wb}{A\text{-}m}$$

where μ_0 is called ~~absolute~~ ~~permittivity~~ Permeability of Free space

$$\therefore dB = \frac{\mu_0}{4\pi}\cdot\frac{I\,dl\sin\theta}{r^2}$$

In vector form, this law is denoted as

$$\vec{dB} = \frac{\mu_0}{4\pi}\cdot I\,\frac{\vec{dl}\times\vec{r}}{r^3}$$

$\frac{\vec{r}}{r}$ is the unit vector in the direction of $\vec{r}$

Therefore magnetic induction due to whole length of conductor carrying current I is given by

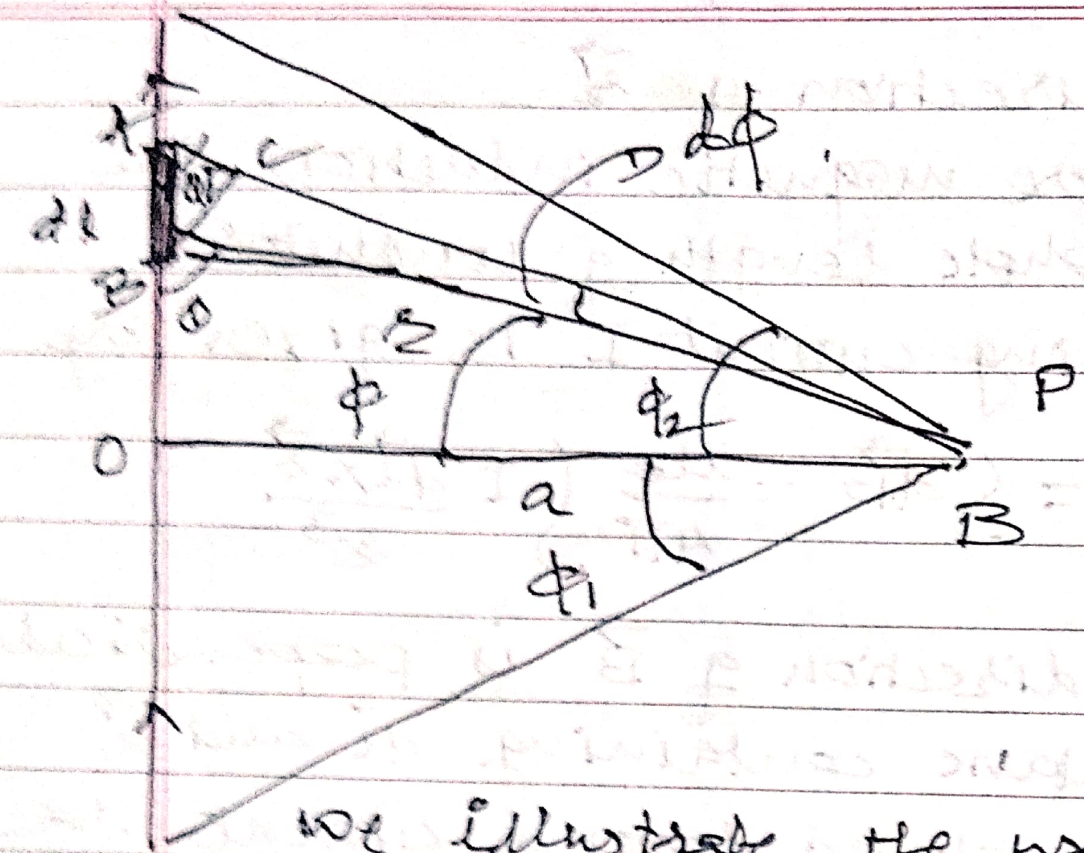
$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi} \int I \frac{d\vec{l} \times \vec{r}}{r^3}$$

the direction of $\vec{B}$ is perpendicular to plane containing $d\vec{l}$ and $\vec{r}$

For closed loop of current the resultant magnetic induction is given by

$$\vec{B} = \frac{\mu_0}{4\pi} \oint I \frac{d\vec{l} \times \vec{r}}{r^3}$$

where the integration is taken over the entire closed loop circuit.



We illustrate the use of Biot–Savart's law by applying it to a calcⁿ of mag. field due to current I in straight wire segment dl. We want to find out mag. field B at a pt. P at a distance a from wire. The center of wire is at origin so pt. P is on the perpendicular bisector of wire.

Magnetic Induction -

The magnetic effect can be produced by a magnet or by a current carrying conductor. The space around a magnet or a current carrying conductor in which the magnetic effect are experienced is called the magnetic field. The basic magnetic field vector is called magne-tic induction. The no. of lines of magnetic induction passing through a unit area placed normal to the lines is defined as magnetic induction or magnetic flux density $\bar{B}$.

The tangent to the line of magnetic induction at any point gives the direction of magnetic induction $\bar{B}$ at that point.

The magnetic induction $\bar{B}$ is also defined as follows

If a test charge q_0 moving with the velocity $\bar{v}$ through a point P in a magnetic field experiences a deflecting force $\bar{F}$ then the magnetic induction at point P is defined by the relation

$$\bar{F} = q_0 \bar{v} \times \bar{B}$$

$$= q_0 v B \sin\theta$$

This relation defines both magnitude and direction of $\bar{B}$. From the above relation we have

$$B = \frac{F}{q_0 v \sin\theta}$$

where θ is the angle between the direction of $\bar{v}$ & $\bar{B}$.

$$\int_{cbed} \vec{B}\cdot d\vec{l} = \vec{B}\int dl$$

$$= \vec{B}x \quad \text{--- (4)}$$

$$B\cdot \cancel{x} = \mu_0 \cdot n\cancel{x}\, I_0$$

$$B = \mu_0 n I_0$$

The M.K.S unit of magnetic induction $\bar{B}$ is weber/m^2 or Tesla or $\frac{\text{Newton}}{\text{Amp. m}}$

If the test charge q_0 is at rest in electric field $\bar{E}$ only the force acting on it is given by.

$$\bar{F} = q_0 \bar{E}$$

if the test charge q_0 is moving in magnetic induction $\bar{B}$ only with the velocity $\bar{v}$, then force acting on it is

$$\bar{F} = q_0\, \bar{v} \times \bar{B}$$

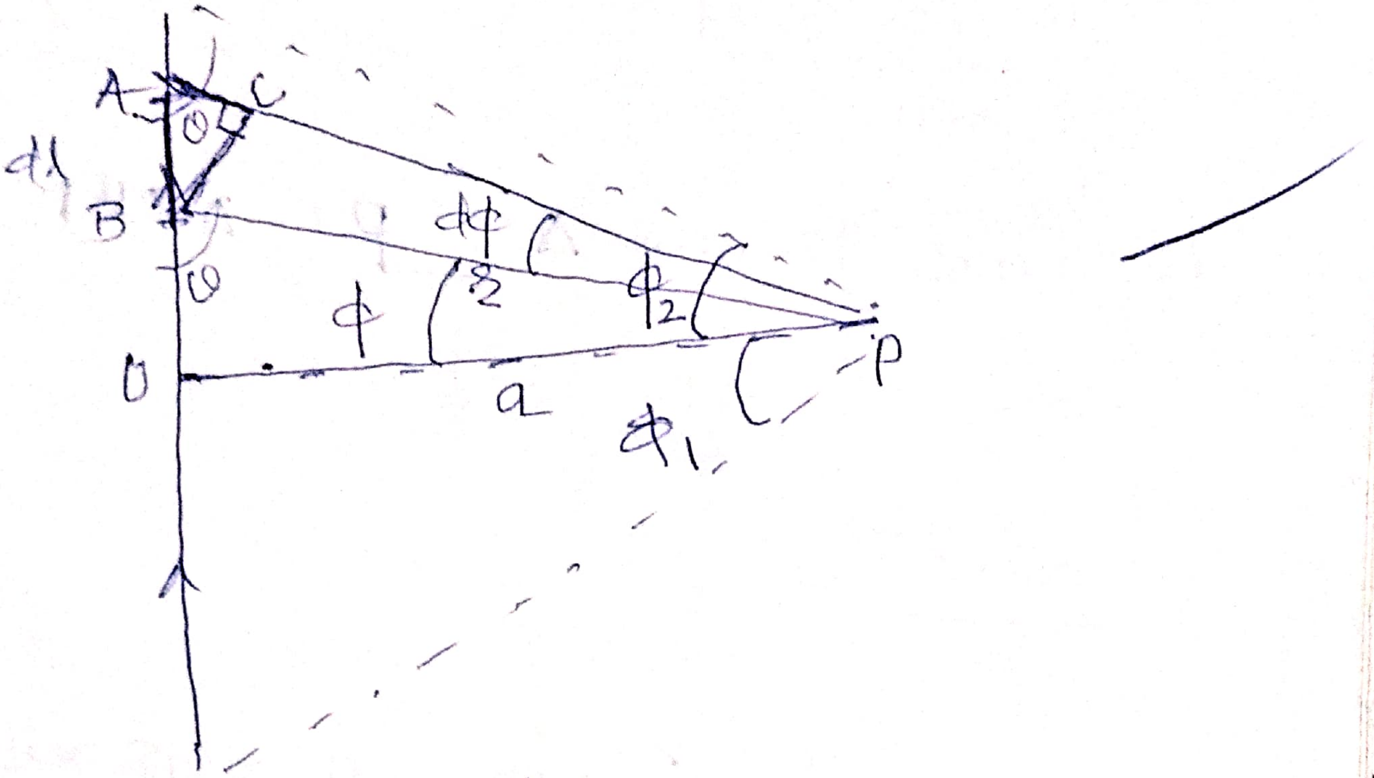
72 - 300 - 4 - 14×6 =
6 - 39×6 = 23

If the test charge q_0 is under the action of electric and magnetic field simultaneously then force acting on it is given by

$$\bar{F} = \cancel{q_0 + C}$$
$$= q_0\bar{E} + q_0 \bar{v}\times\bar{B}$$
$$= q_0(\bar{E} + \bar{v}\times\bar{B})$$

This is called Lorentz force equation.

Magnetic field at a point due to straight conductor carrying current



We know that

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I\,dl \sin\theta}{r^2} \quad \ldots \text{(1)}$$

$$\Delta ABC \quad \sin\theta = \frac{BC}{AB} = \frac{BC}{dl}$$

$$BC = dl \sin\theta.$$

$$\therefore dB = \frac{\mu_0}{4\pi} \frac{I\,BC}{r^2} \quad \ldots \text{(2)}$$

$$d\phi = \frac{\text{Arc}}{r} = \frac{BC}{r}, \quad BC = r\,d\phi$$

$\therefore$ equation (2) becomes

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I \cancel{r} d\phi}{r^{\cancel{2}}}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I d\phi}{r} \quad \ldots\ldots\ldots (3)$$

In the ~~triangle~~ Δ ~~OBP~~ $\angle \Delta OBP$

$$\cos\phi = \frac{OP}{BP} = \frac{a}{r}$$

$$\frac{\cos\phi}{a} = \frac{1}{r}$$

$$r = \frac{a}{\cos\phi}$$ keeping this value

in equation (3) we get.

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I d\phi}{a/\cos\phi}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I d\phi \cdot \cos\phi}{a} \quad \ldots (4)$$

magnetic field at a point P

due to whole conductor.

Integrate equation (4) between

$-\phi_1$ to ϕ_2

$$B=\int_{-\phi_1}^{\phi_2} dB = \frac{\mu_0 I}{4\pi a}\int_{-\phi_1}^{\phi_2} \cos\phi \cdot d\phi$$

$$= \frac{\mu_0 I}{4\pi a} \cdot \left[\sin\phi\right]_{-\phi_1}^{+\phi_2}$$

$$= \frac{\mu_0 I}{4\pi a} \left[\sin\phi_2 - (-\sin\phi_1)\right]$$

$$= \frac{\mu_0 I}{4\pi a} \cdot \left[\sin\phi_2 + \sin\phi_1\right]$$

If the conductor in infinite long $\phi_1 = \phi_2 = 90^\circ$

The magnetic field at point P

16

$$B = \frac{\mu_0 I}{4\pi a}\left[\sin 90^\circ + \sin 90^\circ\right]$$

$$= \frac{\mu_0 I}{4\pi a}\left[1+1\right]$$

$$= \frac{\mu_0 I}{\cancel{4}_{2}\pi a}\cdot\cancel{2}$$

$$= \frac{\mu_0 I}{2\pi a}$$

P

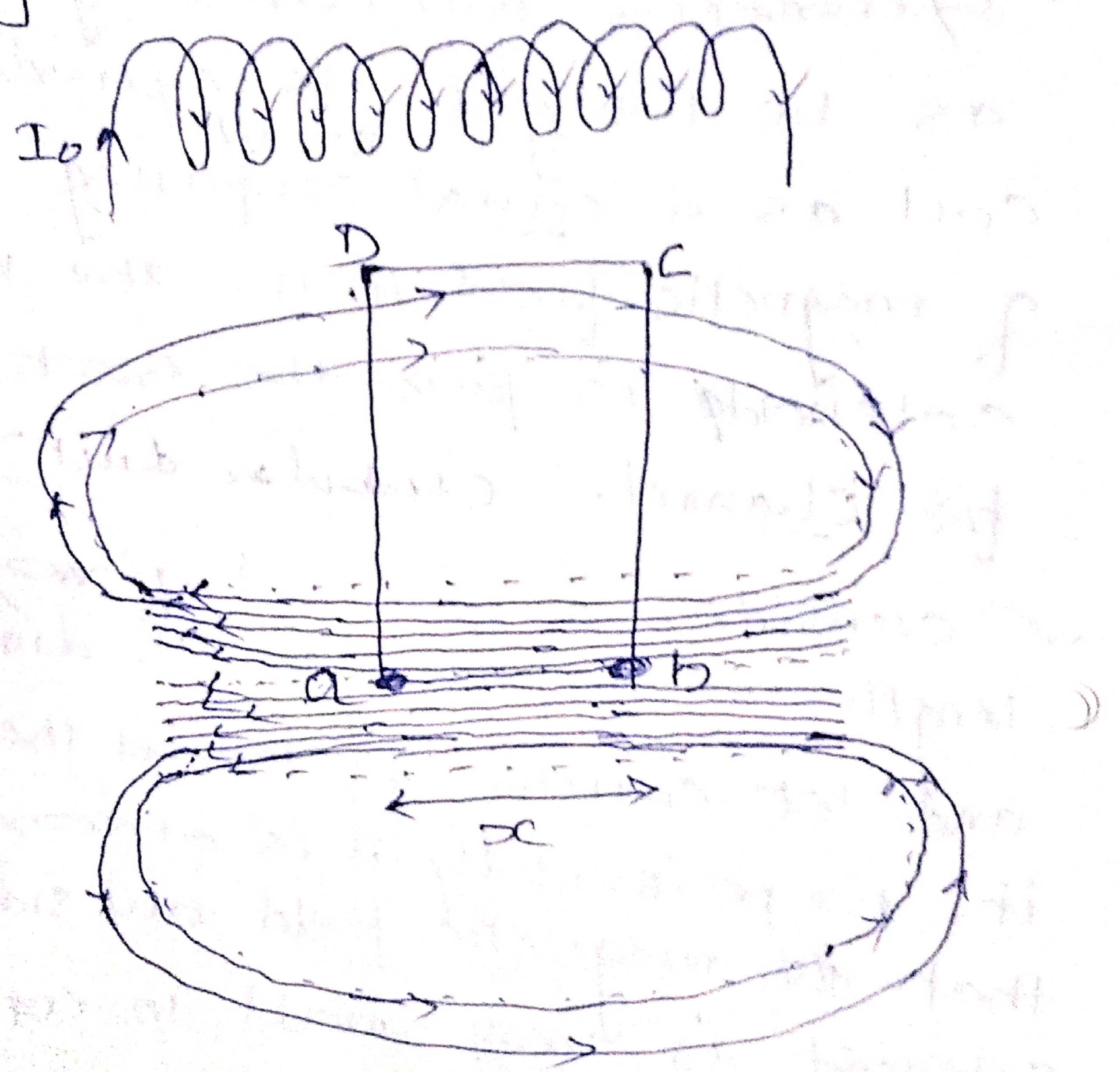
Magnetic induction on the axis of Solenoid

The name Solenoide was first given by Ampere to a wire wounded in a closely spaced spiral over a hollow cylindrical non conducting core, as he thought the cylindrical coil as a canal vessel confining lines of magnetic field in it. The name solenoide is from the Greek word for Channel. (tubular duct)

✓ consider a solenoid of very large length as compared to its diameter and let current I be passed through it. Experimentally it is observed that the magnetic field outside the solenoid is very small in comparison with inside. Also the

Lines of induction are straight and parallel. This implies that the magnetic field is uniform inside the solenoid except near the edges.

Fig.



Consider the Rectangular path abcd. In this figure the side ab is parallel to the axis of solenoid. side 'bc' and 'ad' are taken long enough such that side 'cd' is far away from the solenoid and we can say that at a far dis-tance field due to solenoid is negligible. i.e B at cd is zero

Further for long solenoid the field at ab will be uniform and will be perpendicular to side 'bc' and 'ad'. i.e angle between these sides & B is 90°

Apply Ampere's law to closed path abcd, we have

$$\oint_{abcd} B.dl = \mu_0 I_{enclosed} \quad \text{--- (i)}$$

where,
~~I_0~~ I is the current enclosed by the rectangle abcd and not the total current I_0 flowing through the wire of solenoid.

Let 'n' be the number of turns per unit length of solenoid,
The number of turns in the length ab = x will be equal to nx.
Now the current through each turn is I_0,

Therefore amount of current enclosed in the rectangle will be

$$I = nx I_0 \quad \ldots\ldots (2)$$

The left hand side of the equation ① can be written as

③

$$\oint_{abcd} B\cdot dl = \int_a^b B\cdot dl + \int_b^c B\cdot dl + \int_c^d B\cdot dl + \int_d^a B\cdot dl$$

.........(3)

As side bc & da are perpendi-
-cular to B we have

$$\int_b^c B\cdot dl = \int_b^c B\,dl\cos 90^\circ = 0$$ Also

$$\int_a^d B\cdot dl = \int_a^d B\,dl\cos 90^\circ = 0$$

further B (mag. field) at cd is
zero so that

$$\int_c^d B\cdot dl = 0$$

Hence equation ③ can be written
as

$$\oint_{abcd} B \cdot dl = \int_a^b B \cdot dl$$

Since B is parallel to ab angle between them is zero,

Hence

$$\oint_{abcd} B \cdot dl = \int_a^b B\,dl \cos\theta = \int_a^b B\,dl \cos 0^\circ$$

$\because \cos 0^\circ = 1$

$$= \int_a^b B\,dl = B \int_a^b dl$$

or $\oint_{abcd} B \cdot dl = Bx$ .....(4)

Substituting eqn (4) & (2) in eqn (1) we get

$$B\cancel{x} = \mu_0 (n \cancel{x}) I_0$$

$$B = \mu_0 n I_0 \quad .....(5)$$

equation (6) gives the magnetic field of solenoid at all interior points.

Boit-Savart's law

We illustrate the use of Boit-Savart law by applying it to the calculation of magnetic field due to a current I in the straight wire segment dl. We want to find out mag Field B at a point P at a distance 'a' from the wire. The center of the wire is at origin. so the point P is on the perpendicular bisector of the wire

Ampere's law

Ampere gave a relationship betwee-
-n current (I) and magnetic field $\vec{B}$
which is known as Ampere's law.

This law states that, the line integral of magnetic field $\vec{B}$ around any closed path is equal to μ_0 times the total current (I) passing through that closed path.

$$\therefore \oint \vec{B} \cdot \vec{dl} = \mu_0 I \quad \text{......(1)}$$

The quantity μ_0 represents absolute permitivity of free space or vacuum. If we measure $\vec{B}$ for various distance r and current I in the wire, the experimental results shows that,

$$\vec{B} \propto I, \quad \vec{B} \propto \frac{1}{r} \quad \therefore \vec{B} \propto \frac{I}{r}$$

$$\vec{B} = K\frac{I}{r}$$

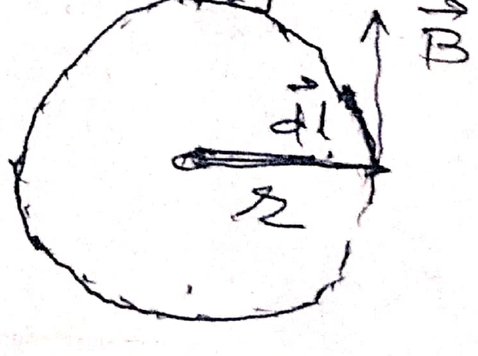
where K is constant of propor-
-tionality and its value is
taken as $\mu_0/2\pi$ hence we
can write

$$\vec{B} = \frac{\mu_0}{2\pi} \cdot \frac{I}{r} \quad \text{------} (2)$$

or $\vec{B}.(2\pi r) = \mu_0 I$ ---(3)

It can be shown that the
term $\vec{B}.(2\pi r)$ is equal to
$\oint \vec{B}.\vec{dl}$.

Consider a path consisting
of a circle of radius r cen-
-tered around wire as
shown in fig. below



2

at every point on the loop, the magnetic field $\vec{B}$ is direc-
-cted along tangential direct-
-ion. Therefore the angle betw-
-een $\vec{B}$ and length element $d\bar{l}$ is zero at all the points.

At each point on the loop the $\bar{B}$ has constant magnitude.

The path is devided into num-
-ber of elements, therefore for each element

$$\vec{B} \cdot d\bar{l} = B\,dl\cos\theta$$

$$= B\,dl$$

$$\because \cos\theta = 1$$

$$\theta = 0$$

the line integral along the closed loop is given by

$$\oint \vec{B} \cdot d\bar{l} = \oint B\,dl\cos\theta$$

$$= \oint B\,dl$$

$$\because \cos\theta = 1$$

$$= B \oint dl \quad \text{------} (4)$$

The total length of all the elements like dl is equal to the circumference of the circular loop

$$\therefore \oint dl = 2\pi r$$

$\therefore$ equation (4) become

$$\oint \bar{B} \cdot \bar{dl} = B \cdot 2\pi r \quad \text{----} (5)$$

According to Amperes law

eqn. (1) $$\oint \bar{B} \cdot \bar{dl} = \mu_0 I$$

$$B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{-----} (6)$$

Ampere's law in differential form

If the current (I) is constituted by a current distribution given by the current density function $\vec{J}$, then

$$I = \iint \vec{J} \cdot d\vec{s} \quad \text{.......} \quad (7)$$

We know $\oint \bar{B} \cdot d\bar{l} = \mu_0 I$ ... (8)

using equation (7) we write above ~~equating (7) & (8) we got~~ equation as follows

$$\oint \bar{B} \cdot d\bar{l} = \mu_0 \iint_S \vec{J} \cdot d\vec{s} \quad \text{———} \quad (9)$$

but according stoke's theorem in vector analysis

$$\oint_l \bar{B} \cdot d\bar{l} = \iint_S \operatorname{curl} \bar{B} \cdot d\bar{s}$$

$$= \iint_S (\bar{\nabla} \times \bar{B}) \cdot d\bar{s} \quad \text{———} \quad (10)$$

From equation ~~⑦ & ⑨~~ 9 & 10, we have

$$\iint_S (\vec{\nabla} \times \vec{B}) \cdot d\vec{s} = \mu_0 \iint_S \vec{J} \cdot d\vec{s}$$

or $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ ———— (11)

The equation (11) is known as Ampere's law in differential form. This equation gives the relation between magnetic field at a point and current density at the same point in space.

Ampere's law in differential form

If the current (I) is constituted by a current distribution given by current density function $\vec{J}$ then

$$I = \iint_S \vec{J} \cdot \vec{ds} \quad \text{———} (1)$$

We know that $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$ ——— (2)

From eqn (1) eqn (2) can be written as

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \iint_S \vec{J} \cdot \vec{ds} \quad \text{———} (3)$$

But according to Stokes Theorem in Vector analysis

$$\oint_l \vec{B} \cdot \vec{dl} = \iint_S \text{curl}\, \vec{B} \cdot \vec{ds}$$

$$= \iint_S (\vec{\nabla} \times \vec{B}) \cdot \vec{ds} \quad \text{———} (4)$$

3 and 4

From eqn ~~and~~ we have

$$\iint_S (\vec{\nabla} \times \vec{B}) \cdot \vec{ds} = \mu_0 \iint_S \vec{J} \cdot \vec{ds}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \quad \text{—— (5)}$$

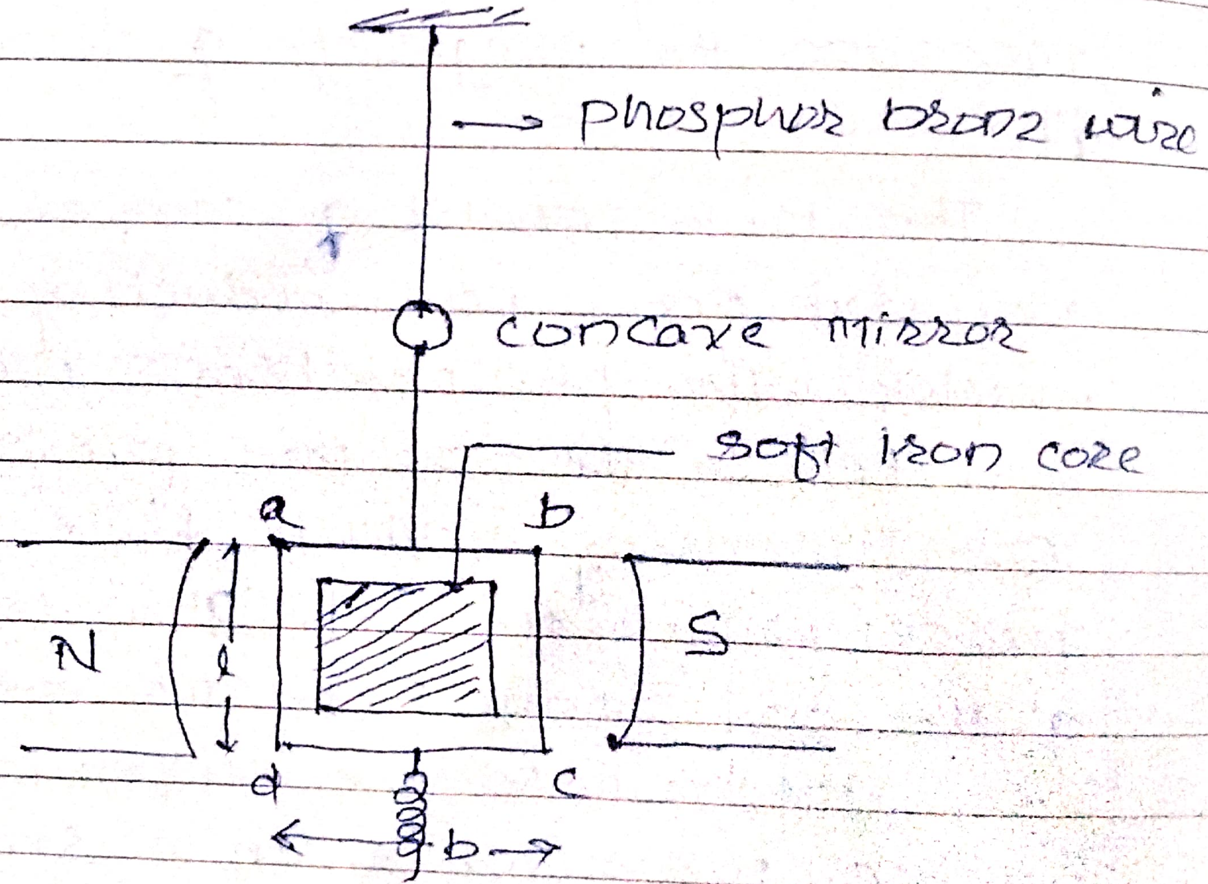
This equation gives Ampere's law in differential form. This equation gives relation between mag field at a point and current density at the same point in space.

Ballistic Galvanometer

The Balistic galvanometer is used for estimating the quantity of charge flowing through it is called B.G. Its working principle is depends on the deflection of coil which is directly proportional to the charge passing through it. The B.G measure the mejority of charge passing through it.

The B.G. consist of copper wire which is wounded on a non conducting frame of galvano-meter. The phosphorbronz wire suspend the coil between the two pole pieces of magnet. For increasing magnetic flux the iron core place within the coil. The lower porsion of the coil connect to the spring. A mirror is attached to the suspension wire which unable to read even a small deflection with the help of lamp and scale arrangement

The M. I of the coil is made large so that the time period of swing of the coil is large that is the coil takes longer time to vibrate in the magnetic field.



Theory of B.G

Let n, l, b and B be the no. of turn of coil l and b be the length and breadth of coil and B the magnetic field in which it is suspended.

Let i be the current at any instant, then Force on each verticle side for very small time $= nlB\,i\,dt$ ———— (1)

Total change in momentum during the time, the total charge passed through it

$$= nlB\int i\,dt$$

$$= nlBq \text{ ———— } (2)$$

$$\because q = \int i\,dt$$

This change in momentum causes a rotation of the coil about axis of suspension producing angular momentum angular mom $= nBq\,l \times b$

$$I\omega = nBqA \quad \text{------- (3)}$$

$l \times b$ = Area of the coil.

Due to angular velocity, coil possess a kinetic energy $\frac{1}{2}I\omega^2$. This kinetic energy producing twist in the suspension wire

$$\frac{1}{2}I\omega^2 = \int_0^{\theta} C\theta\, d\theta$$

$$\frac{1}{2}I\omega^2 = \frac{1}{2}C\theta^2 \Rightarrow I\omega^2 = C\theta^2 \quad \text{------ (4)}$$

where C is restoring twist per unit angular twist.

If T is the period of torsional vibration of the coil, when no current passes through it

$$T = 2\pi\sqrt{I/C}$$

$$T^2 = 4\pi^2 I/C$$

$$I = \frac{T^2 C}{4\pi^2} \quad \text{—— (5)}$$

multiply 4 & 5

$$I^2\omega^2 = \frac{C^2 T^2 \theta^2}{4\pi^2}$$

$$I\omega = \frac{CT\theta}{2\pi} \quad \text{—— (6)}$$

comparing (3) and (6)

$$nBAq = \frac{CT\theta}{2\pi}$$

$$q = \frac{CT\theta}{2\pi nBA}$$

$$q = \frac{T}{2\pi} \cdot \frac{C}{nBA} \cdot \theta \quad \text{—— (7)}$$